Umklapp Processes and the Low-Temperature ($T < 7°K$) Electrical Resistivity of Aluminum*

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Recently, a sharp steplike temperature dependence in the electrical resistivity of aluminum in the vicinity of 4°K has been observed and ascribed to the exponential onset of electron-phonon umklapp scattering. There are, however, theoretical arguments that umklapp processes in aluminum in this temperature region do not account for a sharp temperature dependence of the kind observed. Accurate resistivity measurements (0.02% relative accuracy) are presented in support of these arguments.

During the last few years, many measurements of the low-temperature electrical resistivity of aluminum have been reported.[1–6] One potentially interesting observation is a relatively sharp steplike temperature dependence of the resistivity in the vicinity of 4°K.[1,2] This variation has been ascribed to the exponential onset of electron-phonon umklapp scattering which occurs as the temperature is increased[7] (i.e., the Peierls limitation[8]). Presumably, such behavior would result from a lower limit on the wave vector of the phonons able to take part in the scattering processes. However, there are theoretical arguments that such a mechanism does not account for a sharp temperature dependence of the kind observed in the electrical resistivity of aluminum. We present accurate resistivity measurements to support these arguments.

Consider the Fermi surface of aluminum. Since it intersects both the (111) and (200) Brillouin-

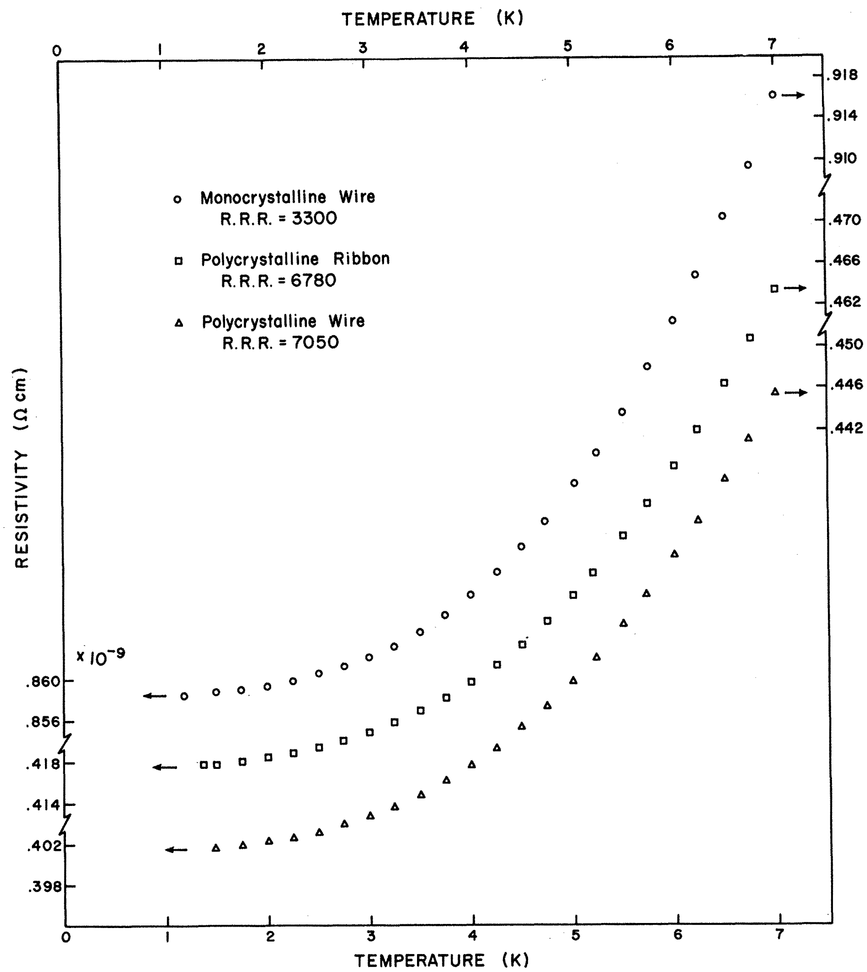

Fig. 1. Electrical resistivity of three aluminum specimens: circle—monocrystalline wire, diameter = 1.3 mm, residual resistivity ratio (RRR) ≡ $\rho_{20°C}/\rho_{0°K} = 3300$; square—polycrystalline ribbon, transverse dimensions: 0.39×1.2 mm, RRR = 6780; triangle—polycrystalline wire, diameter = 1.14 mm, RRR = 7050. The error in the absolute resistivity values, which were determined assuming a resistivity at 20°C of 2.828 $\mu\Omega$ cm and a temperature coefficient near 20°C of 0.00445, is about 2%. The relative error is about 0.02%. Temperatures were measured with an accuracy of 10°mK using a four-terminal germanium resistance thermometer in conjunction with an ac bridge circuit (see Ref. 12).

zone boundaries, umklapp processes involving these reciprocal-lattice vectors will persist down to 0°K. That is, there will be no exponential onset with temperature since there is no lower limit on the wave vector of the phonons taking part in these processes. The next nearest Bragg plane is in the [220] direction; this does not intersect the Fermi surface, but any exponential onset of umklapp processes involving the (2,2,0) reciprocal-lattice vector will not occur until about 200°K.[9] Umklapp processes involving other reciprocal-lattice vectors have still higher onset temperatures. In no case would such a mechanism operate in the 4°K temperature region.

According to a recent theoretical treatment of low-temperature umklapp processes,[10] additional mechanisms involving umklapp scattering which would influence the temperature dependence of the electrical resistivity are: (1) the onset of intraband scattering; (2) the onset of interband scattering; and (3) the partial cutoff of umklapp scattering phase space.[10] Using both the Ashcroft and Heine-Animalu pseudopotentials[11] for aluminum, a two plane-wave calculation of these effects for either pseudopotential led to a smoothly changing temperature dependence of the resistivity in the 1–7°K temperature range. Although the calculations were not in exact agreement with the experimental data, any sharp steplike temperature dependence of the resistivity due to these additional mechanisms was clearly absent.

In Fig. 1 we show the temperature dependence of the electrical resistivity for three reasonably pure aluminum specimens.[13] All specimens measured exhibited a smooth temperature dependence in the 1.2–7°K temperature range. The residual resistance ratios and sizes of the specimens for which results are given in Fig. 1 overlap considerably with the characteristics of specimens in which a sharp steplike temperature variation has been observed.[1,2]

If in fact the sharp temperature dependence were due to the existence of a lower limit on the wave vector of phonons that can participate in umklapp processes, then it should be observable over a wide range of experimental conditions.[14] In particular, the effect should be present in polycrystalline as well as monocrystalline aluminum, and its occurrence should not be affected by the presence of impurity or boundary scattering. Previous observations[1,2] of a sharp change in resistance were limited to selected specimens within a group of specimens, all of which were studied using basically the same technique. Other recent measurements[3–6] have either not covered the relevant temperature range or have not been of sufficient accuracy to affirm or refute the observation of a sharp temperature dependence.

Our experimental results show the smooth variation of resistance with temperature that is expected from a realistic treatment of umklapp scattering processes in aluminum. In particular, the sharp temperature dependence observed previously is not present in our experimental results and is not expected from our theoretical considerations of umklapp processes.

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[1] Yu N. Chiang and V. V. Eremenko, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **3**, 447 (1966) [Soviet Phys. JETP Letters **3**, 293 (1966)].

[2] Yu. N. Chiang, V. V. Eremenko, and O. G. Shevchenko, Zh. Eksperim. i Teor. Fiz. **54**, 1321 (1968) [Soviet Phys. JETP **27**, 706 (1968)].

[3] R. Risnes, Phil. Mag. **21**, 591 (1970).

[4] J. B. Van Zytveld and J. Bass, Phys. Rev. **177**, 1072 (1969).

[5] J. C. Garland and R. Bowers, Phys. Rev. Letters **21**, 1007 (1968).

[6] I. Holwech and J. Jeppesen, Phil. Mag. **15**, 217 (1967).

[7] See, for example, J. M. Ziman, *Electrons and Phonons* (Oxford U. P., Oxford, England, 1967), p. 369.

[8] R. Peierls, Ann. Phys. (Paris) **12**, 154 (1932).

[9] In making this estimate, the lattice constant a was taken to be 4.04 Å, the transverse phonon velocity to be 3×10^5 cm/sec, and r_s to be 2.07 Bohr radii.

[10] For a discussion of these effects and the development of their two plane-wave treatment, see W. E. Lawrence, Ph.D. dissertation, Cornell University, 1970, available from University Microfilms, Ann Arbor, Mich. (unpublished).

[11] N. W. Ashcroft, Phil. Mag. **8**, 2055 (1963); A. O. E. Animalu and V. Heine, *ibid.* **12**, 1249 (1965).

[12] J. W. Ekin and D. K. Wagner, Rev. Sci. Instr. **41**, 1109 (1970).

[13] These specimens were prepared from aluminum obtained from the Cominco Corp., Seattle, Wash.; all were annealed 2 days in air at 300°C followed by gradual cooling and storage at room temperature. Potential leads were spot welded to the samples; in each case the separation between current and potential contacts was greater than five transverse dimensions. Although the use of measuring currents as high as 10 A produced no detectable change in the resistivity, currents were maintained below 4 A for the data presented in the figure.

[14] This would not be the case if the anomaly were, for example, of the type observed by Newbower and Neighbor in gallium which was strongly affected by minute magnetic fields and would therefore be dependent upon extremely low measuring currents for its observation: R. S. Newbower and J. E. Neighbor, Phys. Rev. Letters **18**, 538 (1967).